probabilité continue

I) densité de probabilité $f(t)$ sur $[a,b]$

→ d.d.p

* pour montrer que f est une d.d.p il faut 3 conditions :
  - continue sur $[a,b]$
  - positive sur $[a,b]$
  - $\int_a^b f(t)\,dt = 1$

exemple : mq $f(t) = e^{2t}$ n'est pas une d.d.p sur $[0,2]$:

f est continue et positive sur $[0,2]$.

$\int_0^2 e^{2t}dt = \frac{1}{2}\left[e^{2t}\right]_0^2 = \frac{1}{2}(e^4-1)+1$

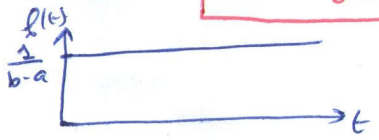
donc f n'est pas une d.d.p.

* Th :

$$p([c,d]) = p(c\leq X\leq d) = \int_c^d f(t)\,dt$$

II) Loi uniforme sur $[a,b]$: (au hasard).

→ d.d.p : $f(t) = \frac{1}{b-a}$



si $[c,d] \subset [a,b]$.

$p(c\leq X\leq d) = p([c,d]) = \int_c^d \frac{1}{b-a}\,dt$

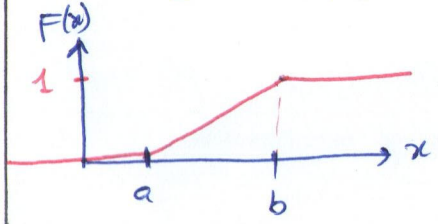
$$p(c\leq X\leq d) = \frac{d-c}{b-a}$$

$$p(X\geq c) = p(c\leq X\leq d) = \frac{d-c}{b-a}$$

$$p(X\leq d) = p(a\leq X\leq d) = \frac{d-a}{b-a}$$

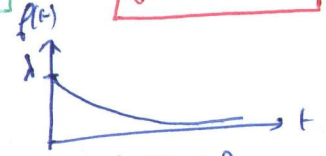
→ fonction de répartition $F(x)$:

$$F(x) \begin{cases} 0 & x<a \\ p(a\leq X\leq x) & \text{si } x\in[a,b] \\ 1 & x>b \end{cases}$$



III) Loi exponentielle de paramètre λ sur $[0,+\infty[$

d.d.p $f(t) = \lambda e^{-\lambda t}$ sur $[0,+\infty[$



si $[c,d] \subset [a,b]$

$p(c\leq X\leq d) = \int_c^d f(t)\,dt = \int_c^d \lambda e^{-\lambda t}dt$

$= -\left[e^{-\lambda t}\right]_c^d = -\left(e^{-\lambda d} - e^{-\lambda c}\right)$

$= e^{-\lambda c} - e^{-\lambda d}$

$$p(c\leq X\leq d) = e^{-\lambda c} - e^{-\lambda d}$$

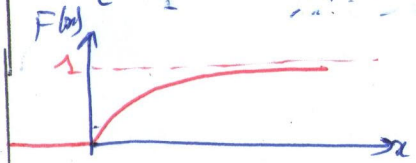
$$p(X\geq c) = p([c,+\infty[) = e^{-\lambda c}$$

$$p(X\leq d) = p(0\leq X\leq d) = 1 - e^{-\lambda d}$$

→ fonction de répartition $F(x)$:

$F : \mathbb{R} \to [0,1]$

$$\begin{cases} 0 & \text{si } x<0 \\ p(0\leq X\leq x) = 1-e^{-\lambda x} & \text{si } x\geq 0 \end{cases}$$



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